On higher-derivative dilatonic gravity
in two dimensions

S. Naftulin

Institute for Single Crystals, 310141 Kharkov, Ukraine

S.D. Odintsov [*]

Department ECM, Faculty of Science, Barcelona University,
Diagonal 647, 08028 Barcelona, Spain

Abstract

We discuss lowering the order of the two-dimensional scalar-tensor R^2 quantum gravity, by mapping the most general version of the model to a multi-dilaton gravity, which is essentially the sigma-model coupled with the Jackiw-Teitelboim-like gravity. In the continuation of our previous research, we calculate the divergent part of the one-loop effective action in a $2D$ scalar-tensor (dilatonic) gravity with the R^2-term, which belongs to a specific degenerate case and cannot be obtained from the general expression. The corresponding finiteness conditions are found.

---

[*]Electronic address: sergei@ecm.ub.es

1   Introduction

Two-dimensional models of gravity are widely recognized to provide a much deeper understanding of the quantum gravitational effects, even beyond the scope of the perturbation theory (for a review, see [1]). Recently there has been a lot of activity in studying the dynamical structure of various models of gravity and their connection to the strings. However, most efforts were undertaken within the conventional (second order in derivatives) models. The dynamical structure of a higher-derivative theory is greatly complicated, even in two space-time dimensions. The most general action of the two-dimensional ($2D$) fourth order scalar-tensor gravity introduced in Ref.[2] contains six different terms with four derivatives:

$$S = \int d^2x \sqrt{-g} \left\{ Z_1(\Phi)(\partial^\mu \Phi \partial_\mu \Phi)^2 + Z_2(\Phi)(\Box \Phi)(\partial^\mu \Phi \partial_\mu \Phi) + Z_3(\Phi)(\Box \Phi)^2 \right.$$
$$\left. + Z_4(\Phi)R\partial^\mu \Phi \partial_\mu \Phi + Z_5(\Phi)R\Box \Phi + Z_6(\Phi)R^2 + \ldots \right\} , \quad (1)$$

where we have suppressed the lower order terms. In general, the potential functions $Z_1, \ldots, Z_6$ are smooth but otherwise arbitrary.

There are several motivations to consider fourth-order theories in two dimensions. First and foremost, they serve as a convenient playground for studying much more intricate four-dimensional models. It is well known that the Einstein gravity is not renormalizable in four dimensions so one often has to resort to the R^2-theory (see, e.g., a book [3] for a comprehensive review); the latter *is* renormalizable, and asymptotically free, but suffers from non-unitarity already at the tree level [4]. In two dimensions, even if the theory does not look unitary, string-inspired arguments may be developed [5] to show that the negative-norm states decouple at the renormalization group (RG) fixed point.Thus, the study of 2D higher derivatives gravities may provide some idea to solve the unitarity problem of such theories in 4D.We show that in formulation with additional scalars there are no problems with massive ghosts as theory maybe mapped to equivalent low derivative model. Second, the simulation data for 2D higher derivative gravity models are getting avaliable [21].Hence, such theories maybe good testing models

of quantum gravity where comparison with numerical datas maybe done.Third, 2D higher derivative terms maybe relevant for problem of the branched polymer [17].Fourth, due to absence of 2D Einstein term there are following possibilities to introduce the kinetic term for graviton-in non-local Polyakov form, in the form of coupling of Einstein term with dilaton or with help of higher derivative term. Note also that in the effective theory of the string with the first massive level taken into account [1] there may also appear higher derivative terms.

Some speculative connection between two- and four-dimensional $(4D)$ theories of gravitation is offered by a dimensional continuation [7]: one starts with the $2 + \epsilon$-dimensional version of quantum gravity and performs the ϵ-expansion, in the end the limit $\epsilon \to 2$ may be taken in order to get some insight into the non-perturbative coupling effects.

Logically, there are two possible points of departure in this construction, namely, the gravity in $4 - \epsilon$ or in $2 + \epsilon$ space-time dimensions. It is reasonable to expect that the properties of both versions have some overlap at intermediate values of ϵ.

Unfortunately, such a matching procedure is not immediate since the origin of the quantum gravity in $2D$ may be very different from that in $4D$: the former is usually considered as the induced quantum gravity (which takes its roots in the string theory) while the latter is not. Hence, if the continuation in ϵ is to be addressed properly one has to take special care about the $4D$ theory to be matched.

Some time ago, an infrared quantum $4D$ gravity was introduced in Ref.[8]: The trace anomaly of matter fields in the curved space-time may be integrated to yield an effective action for the conformal factor of the metric, analogous to the Polyakov action [9] for $D = 2$; the resulting expression describes a fourth-order scalar-tensor theory which seems to be a more suitable candidate for matching across two dimensions $(\epsilon \to 2)$.

There are apparently two ways of descending the $4D$ induced quantum gravity

---

[1]See, e.g., [6] for a recent discussion and further references.

to $D = 2$: the first one is in the parameter space (i.e., by the continuation in ϵ) and the second is by some dimensional reduction. From the first approach one probably concludes that the appropriate $2D$ theory possesses the second order scalar-tensor Lagrangian, while from the second approach it follows that it is of the fourth order. For these two descriptions to be consistent, one has at least to show that a fourth-order action (1) may be re-written as some second-order one. The latter problem is addressed in section 2: making use of a number of auxiliary scalars, Ψ_j, Eq.(1) is converted into the string-inspired form:

$$S = \int d^2x \sqrt{-g} \left[\frac{1}{2} \mathcal{G}_{ab}(\Phi) \partial^\mu \Phi_a \partial_\mu \Phi_b + \mathcal{B}(\Phi) R + \mathcal{U}(\Phi) \right] \, , \qquad (2)$$

where we have denoted $\Phi_a \equiv \{\Phi; \Psi_j\}$. The absence of (perturbative) spin-two states in the $2D$ quantum gravity ensures that the tensor auxiliary fields need not be introduced.

At the risk of belaboring the obvious, let us emphasize the following. The vacuum target manifold of the "string" (2) is essentially curved because of the non-trivial embedding constraints. There is not enough freedom of reparametrization to nullify both non-tachyonic beta-functions, β_B and β_G (in accordance with the observation that the leading terms in the action have dimensions greater than D), thus severe restrictions are to be imposed on the potential functions if the conformal invariance is assumed to hold. This makes a crucial difference with a conventional dilaton gravity where the target-space metric, $\mathcal{G}_{ab}$, is flat and the dilaton, $\mathcal{B}$, is linear in the string co-ordinates so that the possible counterterms are solely due to the tachyon, $\mathcal{U}$, [10].

The one-loop counterterms in the most general version of the fourth-order $2D$ gravity were reported in our earlier work [2]; a discussion of some subtle points connected with the quantum gauge fixing may be also found there. Section 3 below contains the evaluation of divergences in a special (degenerate) class [11] of this theory, which cannot be obtained from the general formulas of Ref.[2]. We also find the family of the dilaton potential functions that guarantee one-loop finiteness. The purely metric R^2-gravity in two dimensions [12], coupled to N

conformal matter scalars,

$$S = \int d^2x \sqrt{-g} \left[\omega R^2 + CR + \Lambda + \frac{1}{2}\partial^\mu \chi_j \partial_\mu \chi_j \right] , \tag{3}$$

belongs to this set: it displays the critical behavior for arbitrary ω, C, and Λ [13]. The divergences are proportional to the Euler characteristic, $\int d^2x \sqrt{g} R$, of the space-time, with the effective central charge $c = 12 - N$. There is a hope that the value of c persists to higher orders in the loop expansion.Note that discussion of generalization of above model for gravitational-torsionful background maybe found in ref.[19].

2 Multi-dilaton gravity from an auxiliary field construction

The first pattern of lowering the order in the higher-derivative gravity was due to Yoneya [12], who showed that the purely metric action (3) could be recast as essentially the JT-like gravity [14], by the use of an auxiliary field, Ψ, representation:

$$[\ldots]^2 = -\Psi^2 - 2\Psi[\ldots] . \tag{4}$$

It is important to realize that, as opposed to the "customary" auxiliary fields, Ψ acquires the kinetic term already at the tree level, due to mixing with the conformal mode via the contact term $\sqrt{g}R\Psi$, [14]. Diagonalizing this, mixed, kinetic matrix one finds that the signs of the eigenmodes are opposite so that the model has zero dynamical degrees of freedom on shell. This agrees with the result of an explicit canonical counting of the degrees of freedom in (3), [12].

Consequently, counting of the degrees of freedom can give us a hint of how many auxiliary scalar fields, Ψ_j, have to be introduced. In our case, (1), the straightforward counting gives two degrees of freedom since the actual difference is one dilaton field, Φ, entering at the fourth order. (Note that in the fourth-order theory, the number of degrees of freedom is effectively doubled [4]

4

as compared to the ordinary one.) So let us firstly try two auxiliary fields, by Yoneya's decomposition (4):

$$S = \int d^2x \sqrt{-g} \left\{ \lambda_1[R + \lambda_2(\partial\Phi)^2 + \lambda_3\Box\Phi]^2 + \lambda_4[\Box\Phi + \lambda_5(\partial\Phi)^2 + \lambda_6R]^2 + \ldots \right\},$$

(Note that similar suggestion to use two scalars for lowering the order in this model has been given recently in ref.[18]). There are six unknown functions, $\lambda_1(\Phi), \ldots, \lambda_6(\Phi)$, to be expressed in terms of six dilatonic "potentials", $Z_1(\Phi), \ldots, Z_6(\Phi)$. It is easily shown that no solution to this algebraic problem exists: the simplest way to proceed is to put $Z_5 = 0$ from the outset, since an arbitrary $Z_5(\Phi)$ can be made zero all the same, by an appropriate conformal metric rescaling (the other Z's also change under such a transformation). In the sigma-model language of Ref.[2] this means that Z_5 defines a Stueckelberg field, [6]. To conclude, two auxiliary fields are not sufficient to lower the order of the derivatives in (1).

This fact is also understood from a trivial observation that the action (1) makes up a quadratic form on "vectors" $V \equiv \lambda_1R + \lambda_2(\partial\Phi)^2 + \lambda_3\Box\Phi$, so one expects to have three independent linear combinations, and correspondingly three auxiliary fields. The only possible reconciliation with the direct counting of the degrees of freedom is that one such a field does not admit a conventional tree-level propagator.

To show this in more detail let us start with the conformal metric rescalings $g_{\mu\nu} \rightarrow g_{\mu\nu}\exp(\sigma)$. Due to the property $\sqrt{g}R \rightarrow \sqrt{g}(R - \Box\sigma)$ different V's mix up under such a transformation. So it is preferable to organize "completing the square" (4) in such a fashion that the respective coefficients do not change, modulo the overall multiplication by $\exp(-\sigma)$. Fortunately, two such functions are known: these are the principal minors in the fourth-order kinetic matrix for the system (1), viz., Z_6 and $\Delta \equiv 4Z_3Z_6 - Z_5^2$ (see Ref.[2] for details).

The preferred way of re-grouping is obvious from the following chain:

$$S = \int d^2x \sqrt{-g} \left\{ -Z_6\Psi_1^2 - 2Z_6\Psi_1 \left[R + \frac{Z_4}{2Z_6}(\partial\Phi)^2 + \frac{Z_5}{2Z_6}\Box\Phi \right] \right.$$

$$+ \left[Z_1 - \frac{Z_4^2}{4Z_6} \right] (\partial\Phi)^4 + \left[Z_2 - \frac{Z_4 Z_5}{2Z_6} \right] (\Box\Phi)(\partial\Phi)^2$$

$$+ \left[Z_3 - \frac{Z_5^2}{4Z_6} \right] (\Box\Phi)^2 + \dots \Bigg\} \tag{5}$$

$$= - \int d^2x \sqrt{-g} \left\{ Z_6 \Psi_1^2 + 2Z_6 \Psi_1 \left[R + \frac{Z_4}{2Z_6}(\partial\Phi)^2 + \frac{Z_5}{2Z_6}\Box\Phi \right] \right.$$

$$+ \frac{\Delta}{4Z_6}\Psi_2^2 + \frac{\Delta}{2Z_6}\Psi_2 \left[\Box\Phi + \frac{2Z_2 Z_6 - Z_4 Z_5}{\Delta}(\partial\Phi)^2 \right]$$

$$\left. + \frac{\Xi}{4Z_6\Delta}\Psi_3^2 + \frac{\Xi}{2Z_6\Delta}\Psi_3(\partial\Phi)^2 - \dots \right\} . \tag{6}$$

Here we have introduced the function

$$\Xi(\Phi) = \left(4Z_1 Z_6 - Z_4^2 \right) \Delta - \left(2Z_2 Z_6 - Z_4 Z_5 \right)^2 . \tag{7}$$

It is a trivial matter to verify that $\Xi \to \Xi \exp(-4\sigma)$ under the conformal transformations. After a few integrations by parts one arrives at the structure (2). As anticipated, the field Ψ_3 does not have a full-fledged tree propagator and its derivatives contribute exclusively to vertices.

As an aside, let us note that the different versions of the general model (1) can be classified into several sets, depending on whether the functions $Z_6(\Phi)$, $\Delta(\Phi)$, or $\Xi(\Phi)$ are zeroes or not. No conformal metric rescaling or local dilaton field re-definition may cause either of them vanish.

Equation (6) defines a map of the general higher-derivative scalar-tensor gravity (1) to a four-dilaton version of gravity (2) with strong sigma-model motives [10, 15].

3 One-loop effective action in dilaton gravity with R^2-term

In this section, we will show that equivalent low derivative model maybe very useful to do the quantum calculations in the situation when the calculations in

terms of original model are extremely difficult to do.In particulary, we complete the study of one-loop divergences in the scalar-tensor higher-derivative gravity (1) initiated in [2].The degenerate case of this theory will be considered as an example. Consider the following action [11]:

$$S = \int d^2x \sqrt{-g} \left[\frac{1}{2} Z(\Phi) \partial^\mu \Phi \partial_\mu \Phi + C(\Phi) R + V(\Phi) + \omega(\Phi) R^2 \right.$$
$$\left. - \frac{1}{2} f(\Phi) \partial^\mu \chi_j \partial_\mu \chi_j \right] , \qquad (8)$$

where we have added N real conformal scalar fields χ_j. All the functions of the dilaton, Φ, are assumed to be analytic. Note that, in principle, an arbitrary dilaton-curvature coupling, $C(\Phi) R$, can be reduced to the linear one, ΦR, by an appropriate redefinition of the Φ-field. However, we do not take this option since it would not facilitate our analysis much.

Basically, the model (8) belongs to a different class from that discussed in the previous section and in Ref.[2] since it has $\Xi, \Delta = 0$. An independent calculation is needed in order to find the divergent structure of the model (8) which is very difficult to do. We will show how this calculation maybe done in terms of low derivative model what is used to restore the one-loop effective action in original theory.

Since the above action contains higher derivatives, which is difficult to deal with, we prefer to lower the order of derivatives in the metric sectors by introducing an auxiliary scalar

$$\Psi = 2\omega R . \qquad (9)$$

The action under consideration becomes:

$$S = \int d^2x \sqrt{-g} \left\{ \frac{1}{2} Z g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + [C + \Psi] R + V - \frac{1}{4\omega} \Psi^2 \right.$$
$$\left. - \frac{1}{2} f g^{\mu\nu} \partial_\mu \chi_j \partial_\nu \chi_j \right\} . \qquad (10)$$

Within the background field formulation, we use the conformal parametrization of the metric fluctuations,

$$g_{\mu\nu} \rightarrow [\exp \sigma] g_{\mu\nu} , \qquad (11)$$

7

and the linear splitting for the scalars

$$\Phi \to \Phi + \varphi \ , \qquad \Psi \to \Psi + \psi \ , \qquad \chi_k \to \chi_k + \eta_k \ . \tag{12}$$

The ghost contribution which corresponds to (11), yields Polyakov's term for the conformal anomaly,

$$\frac{1}{4\pi\epsilon} \int d^2x \sqrt{-g} \frac{13}{3} R + \text{finite terms}, \qquad \epsilon \to +0 \ . \tag{13}$$

Expanding (10) in powers of the quantum fluctuations, $\{\varphi; \psi; \sigma; \eta_k\}$, one obtains:

$$S^{(2)}{}_{,ij} = -\widehat{K}_{ij}\Box + \widehat{L}_{ij}^{\lambda}\nabla_{\lambda} + \widehat{M}_{ij} \ , \tag{14}$$

where

$$\widehat{K} = \begin{pmatrix} Z & 0 & C' & 0 \\ 0 & 0 & 1 & 0 \\ C' & 1 & 0 & 0 \\ 0 & 0 & 0 & -f\delta_{jk} \end{pmatrix} \ , \tag{15}$$

and

$$\begin{aligned} &\widehat{L}_{\varphi\varphi}^{\lambda} = -Z'(\partial^{\lambda}\Phi) \ , \qquad \widehat{L}_{\sigma\varphi}^{\lambda} = -2C''(\partial^{\lambda}\Phi) \ , \\ &\widehat{L}_{\varphi\eta_k}^{\lambda} = -\widehat{L}_{\eta_k\varphi}^{\lambda} = -f'(\partial^{\lambda}\chi_k) \ , \qquad \widehat{L}_{\eta_j\eta_k}^{\lambda} = f'(\partial^{\lambda}\Phi)\delta_{jk} \ , \\ &\widehat{M}_{\varphi\varphi} = C''R - Z'(\Box\Phi) - \frac{1}{2}Z''(\partial\Phi)^2 - \frac{1}{2}f''(\partial\chi_k)^2 + V'' - \left(\frac{1}{4\omega}\right)''\Psi^2 \ , \\ &\widehat{M}_{\varphi\psi} = \widehat{M}_{\psi\varphi} = \frac{\omega'}{2\omega^2}\Psi \ , \qquad \widehat{M}_{\psi\psi} = -\frac{1}{2\omega} \ , \qquad \widehat{M}_{\psi\sigma} = \widehat{M}_{\sigma\psi} = -\frac{1}{2\omega}\Psi \ ; \end{aligned} \tag{16}$$

other relevant matrix elements are zeroes.

Employing the Schwinger-DeWitt technique[2] and adding (13), we finally get the divergent contribution to the one-loop effective action, Γ, for (10):

$$\Gamma_{div} = \frac{1}{4\pi\epsilon} \int d^2x \sqrt{-g} \Bigg\{ \left[\frac{23-N}{6} + \frac{C''}{Z}\right] R - \left[\frac{1}{\omega} + \frac{C'\omega'}{\omega^2 Z}\right] \Psi - \frac{(1/\omega)''}{4Z}\Psi^2$$

---

[2]The technical details of such a calculation in a specifically two-dimensional setting may be found in [16].

8

$$+\frac{V''}{Z} - \frac{C'^2}{2\omega Z} + \left[\frac{N f'^2}{4f^2} - \frac{3Z'^2}{4Z^2} + \frac{Z''}{2Z}\right] g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$$

$$-\left[\frac{f''}{2Z} - \frac{f'^2}{2fZ}\right] g^{\mu\nu}\partial_\mu\chi_j\partial_\nu\chi_j \Bigg\} \quad (17)$$

(plus non-essential surface terms). The constraint (9) might be used to eliminate the auxiliary field Ψ, (in a standard background field method way, see Ch.17 in ref.[20]) which brings about an unexpected dimension-four operator R^2: the latter is then removed by the equation of motion $\delta S/\delta\Phi = 0$. (An alternative approach would be to directly study the sigma-model beta-functions on the target-space background dictated by the specific embedding (9), (10).) That gives the result coinciding with above one on mass shell.

Following the conventional route, one can derive the generalized RG equations: $df/dt = (f''f - f'^2)/2fZ$, etc. However, these equations are not very informative because of their cumbersome, non-linear, structure. Nevertheless, the fixed points of the RG-flow at one loop follow from the conditions of finiteness[3] and can be read off Eq.(17), after the elimination of the auxiliary field Ψ:

$$\frac{11-N}{6} + \frac{C''}{Z} - 2\frac{C'\omega'}{\omega Z} = 0 \ , \qquad (1/\omega)'' = 0 \ , \qquad V'' - \frac{C'^2}{2\omega} = 0 \ , \quad (18)$$

$$\frac{N f'^2}{4f^2} - \frac{3Z'^2}{4Z^2} + \frac{Z''}{2Z} = 0 \ , \qquad \frac{f'^2}{f} - f'' = 0 \ . \quad (19)$$

These equations are most easily solved for $N = 11$, to give

$$f(\Phi) = \alpha_1 \exp\left(\alpha_2\Phi/\sqrt{N}\right) \ , \qquad Z(\Phi) = \frac{\alpha_3 \exp(\alpha_2\Phi)}{\left[\alpha_4 \exp(2\alpha_2\Phi) + 1\right]^2} \ , \qquad (20)$$

$$\omega(\Phi) = (\alpha_5\Phi + \alpha_6)^{-1} \ , \qquad \alpha_5 \neq 0 \ , \qquad (21)$$

$$C(\Phi) = \frac{2\alpha_7}{\alpha_5\Phi + \alpha_6} + \alpha_8 \ , \qquad V(\Phi) = \frac{\alpha_7^2}{\alpha_5\Phi + \alpha_6} + \alpha_9\Phi + \alpha_{10} \ , \qquad (22)$$

[3]The only subtle point that may show up at higher loops is of the reparametrization invariance of the cutoff.

where $\alpha_1, \ldots, \alpha_{10}$ are arbitrary constants. The above restrictions are very stringent: notably, the $2D$ successor [11] of the constant-curvature-constrained model[4] does not belong to this family and hence cannot be described in terms of critical strings.

For $\alpha_5 = 0$, i.e., $\omega \equiv 1/\alpha_6 = const$ another set of solutions exists:

$$C(\Phi) = 2\alpha_7\Phi + \alpha_8 \ , \qquad V(\Phi) = \frac{\alpha_7^2}{\omega}\Phi^2 + \alpha_9\Phi + \alpha_{10} \ , \qquad (23)$$

while $Z(\Phi)$ and $f(\Phi)$ are the same as before. Yoneya's model (3) corresponds to $f, \omega, Z = const$ (in which case Φ is just another matter scalar) and $\alpha_7 = \alpha_9 = 0$. One finds that Yoneya's model coupled to N matter scalars admits the Virasoro algebra with the effective central charge $c = 12 - N$. This value of c is in a perfect agreement with the considerations of Ref.[17] where the "string susceptibility" for (3) in the sigma-model representation was found.

In summary, we have discussed the structure of the higher-derivative $2D$ dilatonic gravity in terms of a second-derivative model with additional scalars. Such a consideration suggests the way to map the string theory in the background of massive modes to the standard sigma-model in a curved (target) spacetime.

Acknowledgements

References

---

[4]In four dimensions this model offers a way to resolve the unitarity problem since the negative-norm spin-two states are modded out.

[1] J.A. Harvey and A. Strominger, in: *Recent Developments in Particle Theory.*, Proceedings of TASI, Boulder, 1992, eds. J.A. Harvey and J. Polchinski (World Scientific, Singapore, 1993).

[2] E. Elizalde, S. Naftulin, and S.D. Odintsov, *Phys. Lett.* **B323**, 124 (1993).

[3] I.L. Buchbinder, S.D. Odintsov, and I.L. Shapiro, *Effective action in quantum gravity.* (IOP, Bristol and Philadelphia, 1992).

[4] K. Stelle, *Phys. Rev.* **D16**, 953 (1977).

[5] I. Antoniadis, C. Bachas, J. Ellis, and D.V. Nanopoulos, *Nucl. Phys.* **B328**, 117 (1989).
R.C. Myers, *Phys. Lett.* **B199**, (1987).

[6] I.L. Buchbinder, E.S. Fradkin, S.L. Lyakhovich, and V.D. Pershin, *Phys. Lett.* **B304**, 239 (1993).
I.L. Buchbinder, V.A. Krykhtin, and V.D. Pershin, Preprint TSPI–TH1/94 (1994).

[7] S. Weinberg, in: *General Relativity: An Einstein Centenary Survey.*, eds. S.W. Hawking and W. Israel (Cambridge University Press, 1979).
R. Gastmans, R. Kallosh, and C. Truffin, *Nucl. Phys.* **B133**, 417 (1978).
S.M. Christensen and M.J. Duff, *Phys. Lett.* **B79**, 213 (1978).
H. Kawai and M. Ninomiya, *Nucl. Phys.* **B336**, 115 (1990). I. Jack and D.R.T. Jones, *Nucl.Phys.* **B358**, 695(1991).

[8] I. Antoniadis and E. Mottola, *Phys. Rev.* **D45**, 2013 (1992).
I. Antoniadis, P.O. Mazur, and E. Mottola, *Nucl. Phys.* **B388**, 627 (1992).
S.D. Odintsov, *Z. Phys.* **C54**, 531 (1992).

[9] A.M. Polyakov, *Phys. Lett.* **B103**, 207 (1981).
R.J. Reigert, *Phys. Lett.* **B134**, 56 (1984).

[10] A.H. Chamseddine, *Phys. Lett.* **B256**, 379 (1991); *Nucl. Phys.* **368**, 98 (1992).
J. Russo and A.A. Tseytlin, *Nucl. Phys.* **B382**, 259 (1992).

[11] I.M. Lichtzier and S.D. Odintsov, *Mod. Phys. Lett.* **A6**, 1953 (1991).
T. Muta and S.D. Odintsov, *Progr. Theor. Phys.* **90**, 247 (1993).

[12] T. Yoneya, *Phys. Lett.* **B149**, 111 (1984).

[13] E. Elizalde, S. Naftulin, and S.D. Odintsov, *Int. J. Mod. Phys.* **A9** 933 (1994).

[14] C. Teitelboim, *Phys. Lett.* **B126**, 41 (1983).
C. Teitelboim, in: *Quantum Theory of Gravity.*, ed. S. Christensen (Hilger, Bristol, 1984) p.327; R. Jackiw, *ibidem*, p.403.

[15] R.B. Mann, **Phys. Rev. D47**, 4438 (1993); *Nucl. Phys.* **B418**, 231 (1994).

[16] S.D. Odintsov and I.L. Shapiro, *Phys.Lett.* **B263**, 183 (1991); *Int. J. Mod. Phys.* **D1**, 571 (1993).
R. Kantowski and C. Marzban, *Phys. Rev.* **D46**, 5449 (1992).

[17] H. Kawai and R. Nakayama, *Phys. Lett.* **B306**, 224 (1993).
J. Nishimura, S. Tamura, and A. Tsuchiya, *Mod. Phys. Lett.* **A9**, 3565 (1994).

[18] I.L. Shapiro, *hep-th* 9501121.

[19] I. Volovich, *Mod.Phys.Lett.* **A8**, 1827 (1993).
W. Kummer and D. Schwarz, *Nucl.Phys.* **B382**, 171 (1992).

[20] P. West, *Introduction to supersymmetry and supergravity*, World Scientific, (1986).

[21] D.V. Boulatov and V.A. Kazakov, *Phys.Lett.* **B184**, 247 (1987); J. Amb-jorn, J. Jurkiewicz and C. Kristiansen, *Nucl.Phys.* **B393**, 601 (1993); S. Ichinose, N. Tsuda and T. Yukawa, KEK preprint (1995).